FERMAT'S LAST THEOREM AND ITS ANOTHER PROOF

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ABSTRACT. We announce here that Fermat's Last theorem was solved, but there is an easy proof of it on the basis of elemetary undergraduate mathematics. We shall disclose such an easy proof and discuss more about it.

1. Introduction

It is a well known fact that Fermat's Last theorem was solved by Andrew Wiles(1953.4.11 -), but his paper is lengthy and hard. So probably many mathematicians still want an easy one even for undergraduates or civilians to understand without difficulty.

This paper could be a sort of an answer to their request. Hence it is hoped that this paper could be really helpful for them.

Of course there could exist papers easier than this paper,

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for example the readers might surf on the arxiv. org to view many uptodate papers including even 1-page paper proving the Fermat Last Theorem.

But we found out therein some serious flaws or gaps before reaching the final goal.

In this paper we shall proceed in the following order.

First, we are going to explain what the Fermat's Last theorem is.

Secondly, we shall give an easy solution even for undergraduates or ordinary people to comprehend.

Finally we finish this paper with a concluding remark dealing with some Diophantine problem which is equivalent to the Fermat's Last theorem .

2. What is the Fermat's Last theorem?

Let us consider the quadratic equation $X^2 + Y^2 = Z^2$ defined in the set of natural numbers. It has a solution X = 3, Y = 4, Z = 5 as an example. As is well known, this equation is obtained from the Pythagorus theorem.

Here we ask a question. What about the equation

$$(2.1) X^n + Y^n = Z^n$$

for any integer value n > 2 defined in the ring of integers \mathbb{Z} ?

Does it have a solution at all with $X \neq 0, Y \neq 0, Z \neq 0$? The answer is 'No' according to the French mathematician Pierre de Fermat(1601.8.17- 1665.1.12).

But his proof was not given except that he said

" I have discovered a truly remarkable proof of this theorem which this margin is too small to contain."

In 1995 the first proof was given by Andrew Wiles. His proof uses the Taniyama-Shimura conjecture relating to elliptic curves and modular forms of them, and his paper is thought to be hard and long.

We intend to give a short and easy proof for this problem by making use of reduction to absurdity.

It suffices to consider only positive integer solutions for this equation.

We may also assume that n is any fixed odd prime number because the exponent has a unique factorization of prime numbers and $X^4 + Y^4 = Z^4$ was solved by Ferma himself. Furthermore we may assume that X, Y, and Z are relatively prime.

If n is a prime number , then one of X and Y cannot be a multiple of n by our assumption.

Suppose that we have a solution for the equation (2.1). If we divide both sides by X^n which may be assumed to be no multiple of n without loss of generality, we get the equation of the form $1 + (Y/X)^n = (Z/X)^n$.

Now if we put

A = Y/X, 1 + B = Z/X, where A and B are rational numbers in the field \mathbb{Q} , then there arises an equation of the form

$$(2.2) 1 + A^n = (1+B)^n,$$

where A and B must be positive rational numbers. Of course A must be exactly greater than B for the equality of (2.2) to hold,i.e., A>B>0.

The converse also holds, i.e., if there is a solution of (2.2) in \mathbb{Q} , then there must be a solution of integers for the equation (2.1).

There are infinitely many solutions for n = 1, 2.

For n = 1, there are obviously many solutions.

For n=2, it says nothing but the Pythagorean theorem, and so we consider the equation of the form $1+A^n=(A+\alpha)^n$. We then have $1+A^2=A^2+2A\alpha+\alpha^2$, so that we may get solutions $A=(1-\alpha^2)/2\alpha$ for any rational number α .

For $n \geq 3$, however, there is no solution at all. We shall prove this fact in the next section 3.

3. Proof of the Fermat's last theorem

Now we assume henceforth once and for all that there is a solution in \mathbb{Q} for the equation

$$(3.1) 1 + A^n = (1+B)^n = B^n + \binom{n}{1} B^{n-1} + \dots + \binom{n}{n-1} B + 1,$$

where A and B are positive rational numbers as manifested just above.

Theorem 3.1. We have no solution of the designated equation (2.1).

Proof. We note first thing that the solution of (2.1), if any, may be obtained roughly from the congruence equation

$$(3.2)$$
 $1+A^n\equiv (1+A)^n\equiv 1+A\equiv (1+B)^n\equiv 1+B^n\equiv 1+B$ modulo (n)

as follows.

(3.3) $A = \frac{t}{s}$, B = (bt+ln)/(bs+kn) for some integers b, l, k, s, and t because $A^n \equiv A$ and $B^n \equiv B$ in the multiplicative group \mathbb{Z}_n^{\times} and so in \mathbb{Z}_n .

In particular we may assume without loss of generality that b, s, t are positive integers. We consider b = 1 at first.

Here the fractions of A and B are assumed to be irreducible ones and we would like to know the relationship of these in (3.3) with n in more detail.

As a matter of fact we are dealing here with the localization $S^{-1}\mathbb{Z}$ of the ring of integers \mathbb{Z} at a prime ideal (n), where S denotes a multiplicative set $\mathbb{Z} - (n)$.

At the same time we may consider by the universal mapping property the composite map: $\mathbb{Z} \to S^{-1}\mathbb{Z} \to \mathbb{Z}_n$ of ring homomorphisms, where \mathbb{Z}_n denotes the finite field consisting of n numbers $\{0, 1, 2, \dots, (n-1)\}$.

From (2.2), we see that s + nk must be a multiple of s and the numerator t of A may happen to be a multiple of the numerator t + nl of B, Thus we may put

$$(3.4) \ v(t+ln) = t, s+kn = ws$$

for some integers v and w. So we may have that both t + ln and s + kn are negative integers. We put c = vw for brevity.

On the other hand we have the following equations successively;

$$1 + A^n = (1 + \frac{A}{c})^n \Rightarrow c^n (1 + A^n) = (c + A)^n \Rightarrow c^n \{1 + (v(t + ln)/s)^n\} = \{c + (v(t + ln)/s)\}^n \Rightarrow c^n \{s^n + (v(t + ln))^n\} = (c + A)^n \Rightarrow c^n \{s^n + (v(t + ln))^n\} = (c + A)^n \Rightarrow c^n \{s^n + (v(t + ln))^n\} = (c + A)^n \Rightarrow c^n \{s^n + (v(t + ln))^n\} = (c + A)^n \Rightarrow c^n \{s^n + (v(t + ln))^n\} = (c + A)^n \Rightarrow c^n \{s^n + (v(t + ln))^n\} = (c + A)^n \Rightarrow c^n \{s^n + (v(t + ln))^n\} = (c + A)^n \Rightarrow c^n \{s^n + (v(t + ln))^n\} = (c + A)^n \Rightarrow c^n \{s^n + (v(t + ln))^n\} = (c + A)^n \Rightarrow c^n \{s^n + (v(t + ln))^n\} = (c + A)^n \Rightarrow c^n \{s^n + (v(t + ln))^n\} = (c + A)^n \Rightarrow c^n \{s^n + (v(t + ln))^n\} = (c + A)^n \Rightarrow c^n \{s^n + (v(t + ln))^n\} = (c + A)^n \Rightarrow c^n \{s^n + (v(t + ln))^n\} = (c + A)^n \Rightarrow c^n \{s^n + (v(t + ln))^n\} = (c + A)^n \Rightarrow c^n \{s^n + (v(t + ln))^n\} = (c + A)^n \Rightarrow c^n \{s^n + (v(t + ln))^n\} = (c + A)^n \Rightarrow c^n \{s^n + (v(t + ln))^n\} = (c + A)^n \Rightarrow c^n \{s^n + (v(t + ln))^n\} = (c + A)^n \Rightarrow c^n \{s^n + (v(t + ln))^n\} = (c + A)^n \Rightarrow c^n \{s^n + (v(t + ln))^n\} = (c + A)^n \Rightarrow c^n \{s^n + (v(t + ln))^n\} = (c + A)^n \Rightarrow c^n \{s^n + (v(t + ln))^n\} = (c + A)^n \Rightarrow c^n \{s^n + (v(t + ln))^n\} = (c + A)^n \Rightarrow c^n \{s^n + (v(t + ln))^n\} = (c + A)^n \Rightarrow c^n \{s^n + (v(t + ln))^n\} = (c + A)^n \Rightarrow c^n \{s^n + (v(t + ln))^n\} = (c + A)^n \Rightarrow c^n \{s^n + (v(t + ln))^n\} = (c + A)^n \Rightarrow c^n \{s^n + (v(t + ln))^n\} = (c + A)^n \Rightarrow c^n \{s^n + (v(t + ln))^n\} = (c + A)^n \Rightarrow c^n \{s^n + (v(t + ln))^n\} = (c + A)^n \Rightarrow c^n \{s^n + (v(t + ln))^n\} = (c + A)^n \Rightarrow c^n \{s^n + (v(t + ln))^n\} = (c + A)^n \Rightarrow c^n \{s^n + (v(t + ln))^n\} = (c + A)^n \Rightarrow (c + A)^n \Rightarrow$$

 $(cs + v(t + ln))^n \Rightarrow c|v(t + ln)$, which means that c must divide v(t + ln) = t by (3.4).

It follows that B = (t+ln)/(s+kn) = v(t+ln)/v(s+kn) = v(t+ln)/vws = v(t+ln)/cs = t/cs = d/s, where cd = v(t+ln) for some positive integer d.

Hence we get s+kn = -s and t+ln = -d, so that 2s = -kn and t+d = -ln are obtained respectively.

It follows that

$$(3.5) \ n|s$$

holds since $n \geq 3$,

which is a contradiction to our assumption that X = s should not be a multiple of n.

We may still have another contradiction granting that (3.5) is right.

If so, then we thus obtain that $s^n \neq 1$, $(s + kn) \neq 1$ along with the fact $s^n \neq (s + kn)^n$ (if n is odd).

But then from (3.1) we get at the equation $(s + kn)^n(s^n + t^n) = s^n \{(s + kn) + (t + ln)\}^n$ by substitution of the fractions, so that $(-s)^n + (-t)^n = (-s + t + ln)^n$. Hence in the field \mathbb{Z}_n we have $-s - t \equiv -s + t + ln$, which reults in $2t \equiv 0$ modulo (n), and hence

$$(3.6) \ n|t$$

because $n \geq 3$.

Note that we should have $k \neq 0, l \neq 0, w = -1$ and $v \leq -2$ as integers. At the same time from the congruence relation (3.2), we have $c \equiv 1 \mod (n)$ as a byproduct.

After all we are led to contradictions (3.5) and (3.6) because we assumed that the fraction t/s = A is irreducible.

Now we consider the general case $B = (bt + ln)/(bs + kn) = (t + \frac{\ln}{b})/(s + \frac{kn}{b})$, whose denominator and numerator are negative numbers.

If $s + \frac{kn}{b} = -s$, then we are led to contradiction likewise as above.

Otherwise we have an equation of the form $1 + (t/s)^n = [1 + (d/s')]^n$, where s' is a divisor of s; so s = ks' for some k with d and s' relatively prime.

We may figure out a general proof as follows.

Consider the equality $1 = [(s'+d)/s']^n - (t/s)^n = [(s'+d)/s' - (t/s)][(s'+d)/s')^{n-1} + ((s'+d)/s')^{n-2}(t/s) \cdots + (t/s)^{n-1}]$ and the factorization of the right hand side of this equation gives 0 < (ks'+kd-t)/s < 1 and (s+kd-t)|s.

If we take the reverse of both sides of the last equation, then we have

 $(3.7) 1 = [s/(ks'+kd-t)][s^{n-1}/((ks'+kd)^{n-1}+(ks'+kd)^{n-2}t+\cdots+t^{n-1})].$ If we assume $(ks'+kd-t) \ge 2$, then we meet a contradiction $1 \equiv 0$ modulo (s+kd-t) since (s+kd-t)|s.

Hence we obtain s+kd-t=1, and so $kd\neq 1$. We thus have

$$(3.8)1 = s^n / [(s+kd)^{n-1} + (s+kd)^{n-2}t + \dots + t^{n-1}],$$

which gives rise to

$$(3.9)1=s^n/[(1+t)^n-t^n]$$

by dint of the identity $[(1+t)^{n-1}+(1+t)^{n-2}t+\cdots+t^{n-1}][(1+t)-t]=(1+t)^n-t^n$.

From (3.9) it follows that
$$s^n = (1 + t - kd)^n = (! + t)^n + n(1+t)^{n-1}(-kd) + \cdots + (-kd)^n = (1+t)^n - t^n$$
 holds.

Hence we get

$$(4.0) \binom{n}{1} (1+t)^{n-1} (-kd) + \binom{n}{2} (1+t)^{n-2} (-kd)^2 + \dots + (-kd)^n = -t^n$$
. Dividing both sides of (4.0) by $-nkd$, we obtain

$$(4.1) (1+t)^{n-1} \left[\binom{n}{2} / n \right] (1+t)^{n-2} (-kd) + \dots + \left[(-kd)^{n-1} / n \right] = (t/kd) (t^{n-1}/n).$$

Beware of the fact $kd \geq 2$. So if we take both sides modulo some prime divisor of d, we are led to a contradiction

 $1 \equiv 0$ after all because of the assumption $n \geq 3$.

So we see that the equality s + kd - t = 1 is impossible,

which implies that there is no solution of (2.1) at all.

Hence we have completely proved in this way the Fermat's Last theorem.

Corollary 3.2. We have no solution of the designated equation (2.1) even in the field \mathbb{Q} of rational numbers.

Proof. Immediate consequence of the above theorem. \Box

Corollary 3.3. $(1-a^n)^{\frac{1}{n}}$ for any rational number a with 0 < a < 1 must be an irrational number. Likewise $(1+a^n)^{\frac{1}{n}}$ for any nonzero rational number a must be an irrational number.

Proof. If we divide both sides of (2.1) by \mathbb{Z}^n , then we get

$$(4.2) \ x^n + y^n = 1$$
, where $0 \le x = X/Z \le 1$ and $0 \le y = Y/Z \le 1$.

Because there is no solution of rational number for the equation $x^n = 1 - y^n$, the first assertion for $(1 - a^n)^{\frac{1}{n}}$ is evident on the one hand.

On the other hand if such a curve on the xy-plane met with the line y = ax for any $a \in \mathbb{Q}$, then we would have an equation of the form $x^n(1+a^n)=1$, or equivalently $x^n=(1+a^n)^{-1}$.

But then there is no rational solution x of this last equation by virtue of the theorem 3.1. Hence the latter assertion is also evident.

Corollary 3.4. For any nonzero rational numbers x, y, we have $x^n + y^n \neq x^n y^n$.

Proof. If we suppose $x^n + y^n = x^n y^n$, then we have $(x^n + y^n)/x^n y^n = 1$ for any nonzero rational numbers x, y.

Thus we get $(1/x)^n + (1/y)^n = 1$, which is nothing but of the form (4.2). It is a contradiction.

4. CONCLUDING REMARK

We believe that there may be other ways to solve the Fermat's Last theorem. But this paper's way is considered to be the easiest among them. Furthermore we might wonder if Fermat himself solved his theorem in this way.

Also we think that most undergraduate students majoring in mathematics might understand this paper without difficulty.

Moreover It is a marvelous fact that (2.1) has no solution even in the field \mathbb{Q} of rational numbers, whose fact is just nothing but a corollary mentioned above of the Fermat's last theorem.

As is well known, the equation of (2.1) is one thing of many sorts of the so called 'Diophantine equations'.

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Let's take a look at some Diophantine equation for a moment such as

$$(4.3) X^n \pm Y^n \pm Z^n = 0$$
,

where $X \neq 0, Y \neq 0, Z \neq 0$.

It is not difficult to see that we can transform this equation into an equivalent Diophantine equation with (2.1).

Even if we are not luminaries in this area, we could absorb ourselves in Diophantine equations.

Because the proofs hereby up to now are elementary, we didn't need to cite bibliographies much except for [KY] in that it shows sort of localization of a ring in the background material chapter.

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